IMPROVING CHAOTIC OPTIMIZATION ALGORITHM USING A NEW GLOBAL LOCALLY AVERAGED STRATEGY

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Abstract. Recently chaotic optimization algorithms as an emergent method of global optimization have attracted much attention in engineering applications. Their good performances have been emphasized [1, 2, 3]. In the frame of evolutionary algorithms, the use of chaotic sequences instead of random ones has been introduced by Caponetto and al. [4]. Since their original work, the literature on chaotic optimisation is flourishing. They are used in the scope of tuning method for determining the parameters of PID control for an automatic regulator voltage, or in order to solve economic dispatch problems, or also for engineering design optimization and in many others physical, economical and biological problems. Different chaotic mapping have been considered, combined with several working strategies. The assessments of the algorithms have been done with respect to numerous objective functions in 1, 2 or 3-dimension. In this paper we present an improvement of the COLM (Chaotic Optimization based on Lozi Map) presented in [1], which is based on a new global locally averaged strategy. The simulation results are done with a 2-D objective function possessing hundreds of local minima, in order to test this new method vs. the previous one in very tough conditions. We emphasize an improvement of the optimisation.

1 Improved COLM Method

Chaos theory (the term chaos was coined par Li and Yorke [5]) is recognized as very useful in many engineering applications. An essential feature of chaotic systems is sensitive dependence on initial condition, (i.e. small changes in the parameters or the starting values for the data lead to drastically different future behaviours). Details about analysis of chaotic behavior can be found in [5, 6, 7, 8, 9]. The application of chaotic sequences can be an interesting alternative to provide the search diversity in an optimization procedure. Due to the non-repetition of chaos, it can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities. A novel chaotic approach is proposed in [1] based on Lozi map [6] which is piecewise linear simplification of the Hénon map [10] and it admits strange attractors. It is given by

$$\begin{cases} y_1(k) = 1 - a|y_1(k-1)| + by(k-1) \\ y(k) = y_1(k-1) \end{cases} \tag{1}$$

where k is the iteration number. In this work, the values of y are normalized in the range [0,1] to each decision variable in 2-dimensional space of optimization problem. This transformation is given by

$$z(k) = \frac{(y(k) - \alpha)}{\beta - \alpha}. \tag{2}$$

where $y \in [-0.6418, 0.6716]$ and $[\alpha, \beta] = [-0.6418, 0.6716]$. The parameters used in this work are $a = 1.7$ and $b = 0.5$. Numerical computation leads to the density $d(s)$ of iterated values of $y(k)$ displayed on Fig. 1. In this figure, the density is normalized to 1 over the whole interval $[0, 1]$ i.e.

$$\int_0^1 d(s)ds = 1.$$

The COLM method introduced in [1] is improved by locally averaging the global search, doing few steps of chaotic local search around every point obtained by the chaotic series.

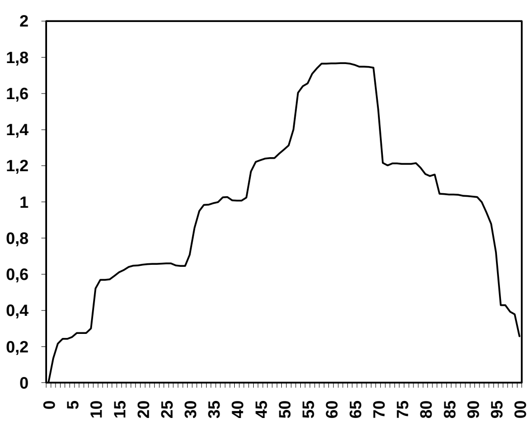

Figure 1: density of iterated values of $y(k)$ of equation (1) over the interval $[0, 1]$ splitted in 100 boxes for 10,000,000,000 iterated values.

Heuristics: the global locally averaged strategy of Improved COLM leads to better results than COLM as shown on Fig. 2. In this figure only three global search results are displayed $x_1(k)$, $x_2(k)$, $x_3(k)$ with

$$f(x_2(k)) < f(x_3(k)) < f(x_1(k)). \tag{3}$$

The local search following global one starts from the best global result $x_2(k)$ (from (3)) and gives $x_2(k+1)$. Instead the local-global search around $x_1(k)$, $x_2(k)$ and $x_3(k)$, leads to $x_1(k+1)$, $x_2(k+1)$, $x_3(k+1)$ which verify

$$f(x_1(k+1)) < f(x_3(k+1)) < f(x_2(k+1)). \tag{4}$$

The local search following the local-global one starts now from the best globally averaged result $x_1(k+1)$ (from(4)) and leads to $\bar{x}$

$$f(\bar{x}) < f(x_1(k+1)). \tag{5}$$

During the chaotic local search, the step λ is an important parameter in convergence behavior of optimization. Hence, two different values of λ are successively employed during the local search. We call this method ICOLM (improved COLM). Many unconstrained optimization problems with continuous variables can be formulated as the following functional optimization problem. Find X to minimize $f(X), X = [x_1, x_2, , x_n]$ Subject to $x_i \in [L_i, U_i]$. Where f is the objective function, and X is the decision solution vector consisting of n variables $x_i \in R^n$ bounded by lower (L_i) and upper limits (U_i).The ICOLM can be illustrated as follows:

Inputs:

M_g: max number of iterations of chaotic Global search.

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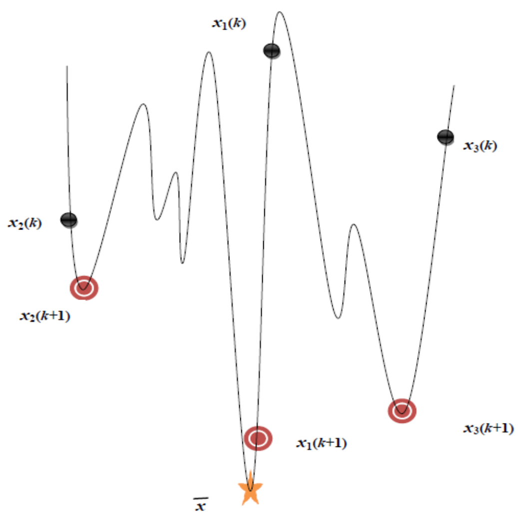

Figure 2: Heuristics of the global locally-averaged strategy.

M_l: max number of iterations of chaotic Local search.
Mgl_1: max number of iter of chaotic Local search in Global search.
Mgl_2: max number of iter of chaotic Local search in Global search.
$Mg \times (Mgl_1 + Mgl_2) + Mg$: stopping criterion of chaotic optimization method in iter.
λ_{gl1}: step size in first global-local search.
λ_{gl2}: step size in second global-local search.
λ: step size in chaotic local search.
Outputs:
$\bar{X}$: best solution from current run of chaotic search.
$\bar{f}$: best objective function (minimization problem).

Step 1 : Initialize the number M_g, M_{gl1},M_{gl2}, M_l of chaotic search and initialization of variables and initial conditions Set k=1, y (0), $y_1(0)$,$a = 1.7$ and $b = 0.5$ of Lozi map. Set the initial best objective function $\bar{f} = +\infty$

- **Step 2**: algorithm of chaotic global search:**while** $k \leq M_g$ **do**

$x_i(k) = L_i + z_i(k).(U_i - L_i)$
if $f(X(k)) < \bar{f}$ **then**
  $\bar{X} = X(k); \bar{f} = f(x(k))$
end if
- **Step 2-1**: sub algorithm of chaotic local search:
while $j \leq M_{gl1}$ **do**
  for $i = 0$ to n **do**
    if $r \leq 0.5$ **then**
      $x_i(j) = \bar{x_i} + \lambda_{gl1} z_i(j).|(U_i - L_i)|$
    else
      $x_i(j) = \bar{x_i} - \lambda_{gl1} z_i(j).|(U_i - L_i)|$
    end if
  end for
  if $f(X(j)) < \bar{f}$ **then**
    $\bar{X} = X(j); \bar{f} = f(x(j))$
  end if
  $j = j + 1$
end while
- **Step 2-2**: sub algorithm of chaotic local search:
while $s \leq M_{gl2}$ **do**
  for $i = 0$ to n **do**
    if $r \leq 0.5$ **then**
      $x_i(s) = \bar{x_i} + \lambda_{gl2} z_i(s).|(U_i - L_i)|$
    else
      $x_i(s) = \bar{x_i} - \lambda_{gl2} z_i(s).|(U_i - L_i)|$
    end if
  end for
  if $f(X(s)) < \bar{f}$ **then**
    $\bar{X} = X(s); \bar{f} = f(x(s))$
  end if
  $s = s + 1$
end while
$k = k + 1$
end while
- **Step 3**: algorithm of chaotic local search:
while $k \leq M_g \times (M_{gl1} + M_{gl2}) + M_l$ **do**
  for $i = 0$ to n **do**
    if $r \leq 0.5$ **then**
      $x_i(k) = \bar{x_i} + \lambda z_i(k).|(U_i - L_i)|$
    else
      $x_i(k) = \bar{x_i} - \lambda z_i(k).|(U_i - L_i)|$
    end if
  end for
  if $f(X(k)) < \bar{f}$ **then**
    $\bar{X} = X(k); \bar{f} = f(x(k))$
  end if
  $k = k + 1$
end while

2 A tough objective function

In order to test this new method vs. the previous one in very tough conditions the simulation results are done with the following 2-D objective function possessing hundreds of local minima: The function f which is very complex, has serval local maxima.

$f = x_1^4 - 7x_1^2 - 3x_1 + x_2^4 - 9x_2^2 - 5x_2 + 11x_1^2x_2^2 + 99sin(71x_1) + 137sin(97x_1x_2) + 131sin(51x_2)$. (6)

We test ICOLM on the search domain: $-10 \leq x_i \leq 10, i = 1, 2$.

The essential feature of this benchmark function is that location of minima is not symmetric. In a forthcoming paper we will extend our numerical analysis in higher dimension with an extended benchmark suite [11].

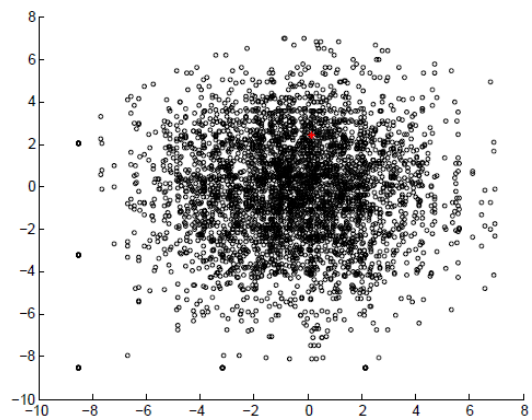

Figure 3: Locally-averaged strategy of chaotic search. Results of every Step 2-2 for f

3 Numerical results

We display few of the results we have obtained showing the better optimization results obtained by this new methods.In each case study, 48 independent runs were made for each of both the COLM and ICOLM methods involving 48 different initial trial conditions $y_1(0)$, $y(0)$ (parameters of Lozi map). not far from the value of f on the global minimum.

For all studied cases, the four configurations, numbered from IC1 to IC4 and C1 ti C4, that are used are presented in tab. 1. The locally averaged strategy of ICOLM is illustrated on Fig. 3 on which the result of every step 2-2 is plotted.

	λ	$\lambda_{M_{gl1}}$	$\lambda_{M_{gl2}}$	M_g	M_l	M_{gl1}	M_{gl2}
IC1	0.001	0.04	0.01	6	50	2	2
IC2	0.01	0.04	0.01	10	50	2	2
IC3	0.1	0.04	0.01	10	50	2	2
IC4	0.1	0.04	0.01	100	50	5	5
C1	0.001			24	50		
C2	0.01			40	50		
C3	0.1			40	50		
C4	0.1			1000	50		

Table 1: The set of parameters values for every run on the benchmark suite defined in Sec. 2. 2.

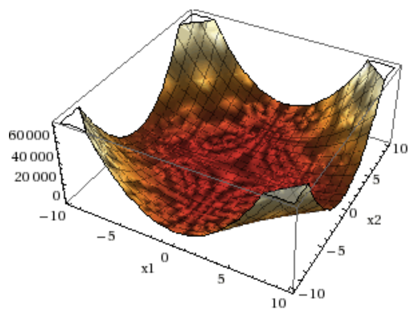

Figure 4: plot of test function used in this study

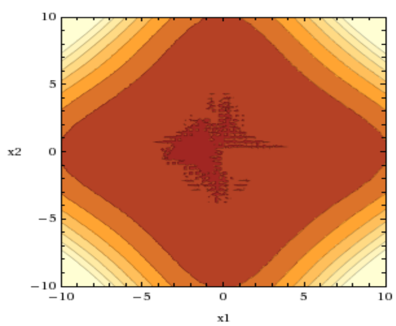

Figure 5: Position of the minima in the search domain

	Best value	Mean value	Std.Dev	$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$
IC1	-384.2891	-363.0185	11.5770	$\begin{pmatrix} -2.7686 \\ -0.4045 \end{pmatrix}$
IC2	-392.5400	-365.7837	14.0615	$\begin{pmatrix} -0.7327 \\ 1.3203 \end{pmatrix}$
IC3	-393.3134	-379.6872	8.8797	$\begin{pmatrix} -1.9677 \\ -1.9982 \end{pmatrix}$
IC4	-395.7338	-381.8734	9.6707	$\begin{pmatrix} -1.8815 \\ -2.2501 \end{pmatrix}$

Table 2: ICOLM

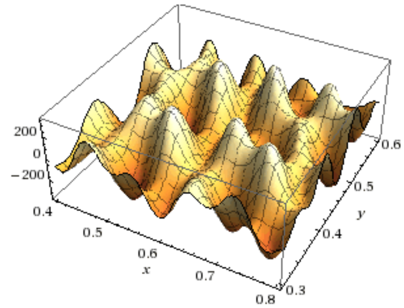

Figure 6: magnification of Fig. 4.

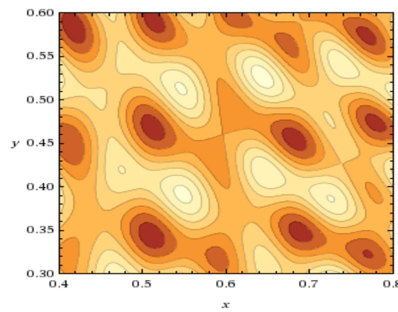

Figure 7: magnification of Fig. 6.

	Best value	Mean value	Std. Dev	$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$
C1	-371.0150	-368.5212	11.1135	$\begin{pmatrix} 0.3105 \\ 0.2442 \end{pmatrix}$
C2	-358.4331	-352.9766	2.4424	$\begin{pmatrix} 3.7283 \\ 6.6158 \end{pmatrix}$
C3	-377.9280	-368.7777	8.8169	$\begin{pmatrix} 3.2738 \\ 6.1685 \end{pmatrix}$
C4	-382.7108	-379.7557	1.7817	$\begin{pmatrix} -5.8930 \\ 2.9309 \end{pmatrix}$

Table 3: COLM

4 Conclusion

In every test, with the same computational cost, ICOLM gives better than COLM best values and Mean Best values but in one case. The presented study allows us to conclude that the proposed method is fast and converges to a good optimum. because we used a sampling mechanism to coordinate the research methods based on chaos theory, and we refined the final solution using a second method of local search. Further research is needed to gain more confidence and better understanding of the proposed methodology. The proposed algorithm has to be evaluated for a large number of test functions in higher dimension.

References

[1] L. S. Coelho. "Tuning of PID controller for an automatic regulator voltage system using chaotic optimization approach", Chaos, Solitons and Fractals, 39, 2009, 1504-1514.

[2] L. S. Coelho. "Reliability-redundancy optimization by means of a chaotic differential evoluytion approach", Chaos, Solitons and Fractals, 41, 2009, 594-602.

[3]D. Davendra, I. Zelinka, R. Senkerik, "Chaos driven evolutionary algorithms for the task of PID control", Computers and Mathematics with Applications, 60, 2010, 1088-1104.

[4]R. Caponetto, L. Fortuna, S. Fazzino and M. G. Xibilia, "Chaotic Sequences to Improve the Performance of Evolutionary Algorithms", IEEE Transactions on Evolutionary Computation, Vol. 7, n 3, June 2003, 289-304.

[5]T. Y. Li and J. A. Yorke, "Period three implies chaos", Amer. Math. Monthly, 82 , 1975, 985-992.

[6]R. Lozi, " Un attracteur étrange? du type attracteur de Hénon ", J. Phys., 39(C5), 1978, 9-10.

[7]S. H. Strogatz, Nonlinear dynamics and chaos, Massachussets: Perseus Publishing, 2000.

[8]T. S. Parker, L.O. Chua, Practical numerical algorithms for chaotic system. Berlin, Germany: Springer; 1989.

[9]] K. T. Alligood, T. D. Sauer, J. A. Yorke, Chaos: an introduction to dynamical systems, London, UK: Springer; 1996.

[10]M. Hénon, "A two dimensional mapping with a strange attractor" Commun. Math. Phys. 1976; 50(1):69-77.

[11]S. Cong, G.Li and X.Feng, "'An Improved Algorithm of Chaos Optimization", Proceedings 2010 8th IEEE International Conference on Control and Automation.